Summary of *Foundations for Success:* The Final Report of the National Mathematics Advisory Panel, 2008

The National Mathematics Panel was charged with defining a system for delivering instruction in mathematics that enables all students to acquire competence in algebra and readiness for higher levels of mathematics. Although the report's findings and recommendations are focused on grades PreK-8, the PreK-8 path laid down by the panel is directed toward the study of *authentic algebra* in middle and secondary school. The panel goes on to define the content of *authentic algebra* in a one-page table of major topics.

The system of mathematics education set forth by the Panel includes Curricular Content, Learning Processes, Teachers and Teacher Education, Instructional Practices, Instructional Materials, Assessments, and Research Policies and Mechanisms. The report's Executive Summary presents 45 major findings and recommendations for the system that are based upon, among other resources, over 16,000 research publications and policy reports.

Curricular Content:
The Panel determined a progression of learning mathematics from less sophisticated topics to more sophisticated ones. For example, by the end of grade 5, students should be proficient, by that they should, understand the important concepts and develop flexible, accurate, and automatic execution of algorithms to solve problems working with whole numbers. Followed by, at the end of grade 7, students should be proficient with fractions, including decimals, percents, and negative fractions. The implementation of such progressions will support student proficiency in algebra with students understanding the important concepts and developing flexible, accurate, and automatic execution of algorithms to solve problems.

Learning Processes:
Since students come to school with different mathematical knowledge, effective mathematics interventions are necessary in every classroom. Curriculum must address the development of conceptual understanding, computational fluency, and problem solving simultaneously. This can be achieved when students are afforded sufficient time to learn. Students and families need to understand the importance of student effort as related to successful mathematics performance.

Teacher and Teacher Education:
Research shows direct relationship between teachers' content knowledge and their students' achievement. Key to improving teachers' effectiveness in the classroom is that teachers engage in multiple opportunities to learn the mathematics for teaching. Teacher education programs must graduate teachers knowledgeable in teaching school mathematics. Schools and district must support teachers throughout their career.

Instructional Practices:
A balance between teacher and student-centered approaches to teaching and learning is critical for student success in learning mathematics. The needs of students whether they are struggling or mathematically talented need to be met. Regular use of formative assessments to design and individualize instruction improves students learning.

Instructional Materials:
Schools and districts need to provide instructional materials aligned with state standards and the recommendations of the National Mathematics Panel and free of mathematical errors with regard to accuracy, clarity, and logical reasoning. Also, publishers should produce much shorter and more focused textbooks.

Assessment:
State and national assessments need to be aligned with the Panel's curricular content recommendations and accessible to all students.

Research Policies and Mechanisms:
Continuous research in partnership with PreK-16 educators needs to be conducted to ensure effective curricular, instructional practices, assessments, and teacher education. This PreK-16 partnership includes educators, psychologists, sociologists, economists, cognitive scientists, and mathematicians.

The National Mathematics Panel identified Benchmarks for Critical Foundations to Algebra. The interpretation of these benchmarks should be flexible to accommodate student needs. Three clusters of concepts and skills that are foundational for formal algebra coursework include:

- Fluency with whole numbers
- Fluency with fractions, and
- Particular aspects of geometry and measurement

Fluency with Whole Numbers:

- By the end of Grade 3, students should be proficient with the addition and subtraction of whole numbers.
- By the end of Grade 5, students should be proficient with multiplication and division of whole numbers.

Fluency with Fractions:

- By the end of Grade 4, students should be able to identify and represent fractions and decimals and compare them on a number line or with other common representations of fractions and decimals.
- By the end of Grade 5, students should be proficient with comparing fractions and decimals and common percents, and with the addition and subtraction of fractions and decimals.
- By the end of Grade 6, students should be proficient with multiplication and division of fractions and decimals.
- By the end of Grade 6, students should be proficient with all operations involving positive and negative integers.
- By the end of Grade 7, students should be proficient with all operations involving positive and negative fractions.
- By the end of Grade 7, students should be able to solve problems involving percent, ratio, and rate, and extend this work to proportionality.

Particular Aspects of Geometry and Measurement:

- By the end of Grade 5, students should be able to solve problems involving perimeter and area of triangles, and all quadrilaterals having at least one pair of parallel sides (i.e. trapezoids).
- By the end of Grade 6, students should be able to analyze the properties of two-dimensional shapes and solve problems involving perimeter and area. They should also be able to analyze properties of three- dimensional shapes and solve problems involving surface area and volume.
- By the end of Grade 7, students should understand relationships involving similar triangles.

The Major Topics of School Algebra

Symbols and Expressions

- Polynomial expressions
- Rational expressions
- Arithmetic and finite geometric series

Linear Equations

- Real numbers as points on the number line
- Linear equations and their graphs
- Solving problems with linear equations
- Linear inequalities and their graphs
- Graphing and solving systems of simultaneous linear equations

Quadratic Equations

- Factors and factoring of quadratic polynomials with integer coefficients
- Completing the square in quadratic expressions
- Quadratic formula and factoring of general quadratic polynomials
- Using the quadratic formula to solve equations

Functions

- Linear functions
- Quadratic functions—word problems involving quadratic functions
- Graphs of quadratic functions and completing the square
- Polynomial functions (including graphs of basic functions)
- Simple nonlinear functions (e.g., square and cube root functions; absolute value; rational functions; step functions)
- Rational exponents, radical expressions, and exponential functions
- Logarithmic functions
- Trigonometric functions
- Fitting simple mathematical models to data

Algebra of Polynomials

- Roots and factorization of polynomials
- Complex numbers and operations
- Fundamental theorem of algebra
- Binomial coefficients (and Pascal's Triangle)
- Mathematical induction and the binomial theorem

Combinatorics and Finite Probability

- Combinations and permutations, as applications of the binomial theorem and Pascal's Triangle